NYU-TH/00/02/02

Metastable Gravitons and Infinite Volume Extra Dimensions

G. Dvali, G. Gabadadze and M. Porrati

Department of Physics, New York University, New York, NY 10003

Abstract

We address the issue of whether extra dimensions could have an infinite volume and yet reproduce the effects of observable four-dimensional gravity on a brane. There is no normalizable zero-mode graviton in this case, nevertheless correct Newton's law can be obtained by exchanging bulk gravitons. This can be interpreted as an exchange of a single *metastable* 4D graviton. Such theories have remarkable phenomenological signatures since the evolution of the Universe becomes high-dimensional at very large scales. Furthermore, the bulk supersymmetry in the infinite volume limit might be preserved while being completely broken on a brane. This gives rise to a possibility of controlling the value of the bulk cosmological constant. Unfortunately, these theories have difficulties in reproducing certain predictions of Einstein's theory related to relativistic sources. This is due to the van Dam-Veltman-Zakharov discontinuity in the propagator of a massive graviton. This suggests that all theories in which contributions to effective 4D gravity come predominantly from the bulk graviton exchange should encounter serious phenomenological difficulties.

If Standard Model particles are localized on a brane, the volume of extra dimensions can be as large as a millimeter without conflicting to any experimental observations [1]. This is also true for warped spaces in which the extra dimensions are non-compact but have a finite volume [2] (for earlier works on warped compactifications see [3, 4]).

In the framework of Ref. [1] the volume $V \sim L^N$ of extra N space dimensions sets the normalization of a four-dimensional graviton zero-mode. Therefore, the relation between the observable and the fundamental Planck scales (M_P and M_{Pf} respectively) reads as follows:

$$M_P^2 \ = \ M_{Pf}^{2+N} V \ . \tag{1}$$

A similar relation holds for the Randall-Sundrum (RS) scenario [2], where the role of L^{-2} is played by the curvature of AdS_5. In this case the extra dimension is not compact, nevertheless its length (or volume) is finite and is determined by the bulk cosmological constant $L \propto 1/\sqrt{|\Lambda|}$. In the scenario of Ref. [1] gravity becomes high-dimensional at distances $r << L$ with the corresponding change in Newton's law

$$1/r \to 1/r^{1+N} \ . \tag{2}$$

The same holds true for RS-type scenarios with non-compact extra dimensions [5].

The purpose of the present letter is to study whether the volume of extra space can be truly *infinite* while the four-dimensional Planck mass is still finite. In this case the relation (1) should somehow be evaded. Since there are no normalizable zero modes in such cases, the effects of 4D gravity must be reproduced by exchanging the continuum of bulk modes. An example of this type was recently proposed in Ref. [6]. The physical reason why such an exchange can indeed mimic the $1/r$ Newtonian law can be understood as follows. The four-dimensional graviton, although is not an eigenstate of the linearized theory, can still exist as a *metastable* resonance with a finite lifetime τ_g. In such a case one might hope that the exchange of this graviton approximates Newton's law at distances shorter than the graviton lifetime but changes the laws of gravity at larger scales. The question is whether the four-dimensional effective theory obtained in this way is phenomenologically viable. In the present paper we will argue that, despite the correct Newtonian limit, the infinite volume scenario has problems in reproducing other predictions of Einstein's theory. This problem is shared by any model in which the dominant contribution to 4D gravity comes from the exchange of continuum states. The reason behind such a discrepancy is the van Dam-Veltman-Zakharov discontinuity in the propagator of a massive spin-2 field in the massless limit [7]. Very briefly, the physical effects of additional polarizations of a massive graviton survive in the massless limit and change dramatically predictions of the theory [7]. As a result, any theory which relies on the exchange of massive (no matter how light) gravitons, will give rise to predictions which differ from those of General Relativity.

We shall consider the five-dimensional Einstein gravity coupled to an arbitrary energy-momentum tensor T_{AB} which is independent of four space-time coordinates

x_μ. We will assume that T_{AB} results from a certain combination of branes, bulk cosmological constant Λ and classical field configurations which preserve four-dimensional Poincaré invariance. Einstein's equations $G_{AB} = T_{AB} + g_{AB}\Lambda$ give rise to a metric of the following form

$$ds^2 \;=\; A(z)\;\left(ds_4^2 - dz^2\right)\;, \tag{3}$$

where z is an extra coordinate, and we assume that the four-dimensional metric ds_4^2 is flat. The volume of the extra space in this construction is determined by an integral

$$\int_{-\infty}^{+\infty} A^{5/2}(z)\;dz\;. \tag{4}$$

For instance, in the RS framework $A(z) \;= (1+H|z|)^{-2}$, where H is proportional to the square root of the cosmological constant $H \propto \sqrt{|\Lambda|}$. This warp factor gives rise to a finite expression in (4). An infinite volume theory is obtained if, for instance, the value of $A(z)$ tends to a *nonzero* constant as z goes to $\pm\infty$. If the volume is finite, there is a normalizable zero-mode graviton in the spectrum of fluctuations about this background [2]. Indeed, let us parametrize the four-dimensional graviton fluctuations as follows:

$$ds^2 = A(z)\left[(\eta_{\mu\nu} + h_{\mu\nu}(x,z))dx^\mu dx^\nu - dz^2\right]\;. \tag{5}$$

The corresponding linearized Schrödinger equation for the excitation $h_{\mu\nu}(x,z) = A^{-3/4}(z)\Psi(z)h^{(0)}_{\mu\nu}(x_\mu)$ takes the form:

$$\left(-\partial_z^2 + \frac{3}{4}\left\{\frac{A''}{A} - \frac{A'^2}{4A^2}\right\}\right)\;\Psi(z)\;=\;m^2\;\Psi(z)\;. \tag{6}$$

Where $\eta^{\mu\nu}\partial_\mu\partial_\nu h^{(0)} = -m^2 h^{(0)}$ and primes denote differentiation with respect to z. This equation has a zero-mode solution for a generic form of a warp factor $A(z)$:

$$\Psi_{\rm zm}(y)\;=\;A^{3/4}(z)\;. \tag{7}$$

This implies that the z dependent part of the fluctuations $h(x,z)$ is just a constant, i.e., $h(x,z) = {\rm const.}\exp(ipx)$. If the integral in (4) diverges, the volume is infinite and the zero-mode is not normalizable. Thus, the spectrum in this case would consist of continuum states only. One should notice, however, that even in this case the correct Newtonian limit may be recovered in some approximation by exchanging the continuum of non-localized states! The physical reason for such a behavior is that the 4D localized graviton, although not an eigenstate, can still exist as a *metastable* resonance with an exponentially long lifetime, τ_g. Therefore, the exchange of this resonance could give rise to a correct Newtonian potential at intermediate distances. Note that this is just a reformulation of the fact that the tower of the bulk states conspire in such a way that the Newtonian potential is seen as an approximation.

To make these discussions a bit precise let us turn to the propagator of a massive metastable graviton. This is given as follows:

$$G_4^{\mu\nu\alpha\beta}(x) = \int \frac{dp^4}{(2\pi)^4} \left(\frac{1}{2}(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}) - \frac{1}{3}g^{\mu\nu}g^{\alpha\beta} + \mathcal{O}(p) \right) D(p^2, m_0^2, \Gamma)e^{-ip(x)}, \tag{8}$$

where $D(p^2,\ m_0^2,\ \Gamma)$ stands for the scalar part of a massive resonance propagator, $D(p^2, m_0^2, \Gamma) = (p^2 - m_0^2 + im_0\Gamma)^{-1}$. Γ denotes the width of the resonance. The momentum dependent part in the tensor structure gives zero contribution when the propagator is convoluted with a conserved energy-momentum tensor, thus, this part will be omitted. In order to make a contact with continuum modes, let us use the following spectral representation for D:

$$D(p^2,\ m_0^2,\ \Gamma)\ =\ \frac{1}{2\pi} \int\ \frac{\rho(s)}{s - p^2 + i\epsilon}\ ds\ , \tag{9}$$

where s denotes the Mandelstam variable and $\rho(s)$ is a spectral density. If the assumption of the resonance dominance is made, then $\rho(s)$ is approximated by a sharply peaked function around the resonance mass $s = m_0^2$. In what follows we assume that the resonance lifetime is very big and we neglect the effects of a nonzero resonance width (these will modify gravity laws at very large distances only). Exchanging such a particle between two static sources one obtains the potential

$$V(r) \sim \int\ \frac{e^{-\sqrt{s}r}}{r}\ \rho(s)\ ds\ . \tag{10}$$

This expressions reproduces a standard $1/r$ interaction at distances $r << (m_0)^{-1}$ in the single, narrow-resonance approximation, $\rho(s) \propto \delta(s - m_0^2)$. On the other hand, we can expand the spectral density $\rho(s)$ into the complete set of bulk modes

$$\rho(s) = \int_0^\infty\ |\psi_m(0)|^2\ \delta\ (s - m^2)\ dm\ . \tag{11}$$

Here, $\psi_m(0)$ denote the wave functions of the bulk modes at the point $z = 0$. Using this expression for the spectral density one finds the following potential

$$V(r)\ \sim\ \int_0^\infty\ \frac{e^{-mr}}{r}\ |\psi_m(0)|^2\ dm\ . \tag{12}$$

This is nothing but the potential mediated by the continuum of the bulk modes. Thus, the effect of the metastable graviton, when it exists, can be read off the expression which includes all the bulk modes. These two descriptions are complementary to each other. In the case when the resonance exists, the continuum modes can conspire in such a way that (12) yields the $1/r$ law in a certain approximation. The inverse statement is also likely to be true. In the appendix we will show explicitly the presence of a resonance state in a model with infinite volume extra dimension

and $1/r$ potential produced by the bulk modes [6]. As we mentioned above, such model gives Newtonian gravity only at intermediate distances. At large distances, the five-dimensional laws of gravity should be restored (due to the metastable nature of the resonance). This phenomenon could have dramatic cosmological and astrophysical consequences. Indeed, at large cosmic scales the time-dependence of the scale factor $R(t)$ in Freedman-Robertson-Walker metric would dramatically change due to the change in the laws of gravity[1].

Another interesting comment concerns bulk supersymmetry. Since the volume of the extra dimension is infinite, it might be possible to realize the following scenario. The bulk is exactly supersymmetric and SUSY is completely broken on a brane (this could be a non-BPS brane which is stable for some topological reasons [9]). The transmission of SUSY breaking from the brane worldvolume to the bulk is suppressed by the volume of the bulk and is vanishing. In such a case one could imagine a setup where the bulk cosmological constant is zero due to the bulk SUSY[2].

Having these attractive features of the theories with truly infinite extra dimensions discussed, we move to some phenomenological difficulties of these models. In fact, we will argue below that these theories cannot reproduce other predictions of Einstein's general relativity. The reason is that all the spin-2 modes that dominantly contribute to the four-dimensional gravity in this case are massive modes. It has been known for a long time [7] that propagator of massive spin-2 states has no continuous massless limit. As a result the effects of the massless spin-2 graviton are different from the massive one, no matter how small the mass is. Let us show how this affects the phenomenology of infinite volume theories. The four-dimensional gravity on a brane is reproduced by an exchange of the continuum of bulk gravitons. At a tree level this gives

$$G_5 \int_0^\infty dm \int dx^4\ d^4x'\ T_{\mu\nu}(x)\ G_m^{\mu\nu\alpha\beta}(x-x')\ T'_{\alpha\beta}(x')\ , \tag{13}$$

where $T_{\mu\nu}(x)$ and $T'_{\mu\nu}(x')$ are the energy-momentum tensors for two gravitating sources. For $m \neq 0$ the graviton propagator is given by

$$G_m^{\mu\nu\alpha\beta}(x-x') = \int \frac{dp^4}{(2\pi)^4} \frac{\frac{1}{2}(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}) - \frac{1}{3}g^{\mu\nu}g^{\alpha\beta} + \mathcal{O}(p)}{p^2 - m^2 - i\epsilon} e^{-ip(x-x')}\ , \tag{14}$$

whereas for $m = 0$ we have

$$G_0^{\mu\nu\alpha\beta}(x-x') = \int \frac{dp^4}{(2\pi)^4} \frac{\frac{1}{2}(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}) - \frac{1}{2}g^{\mu\nu}g^{\alpha\beta} + \mathcal{O}(p)}{p^2 - i\epsilon} e^{-ip(x-x')}\ . \tag{15}$$

As we see, the tensor structures in the two cases are different. In the massless limit, the propagator exhibits the celebrated van Dam-Veltman-Zakharov discontinuity.

[1] A different possibility to modify the long distance gravity due to an additional massive graviton was proposed earlier in Ref. [8]. In the view of phenomenological problems discussed below, this graviton should be very weakly coupled.

[2] Note that local SUSY does not necessarily imply vanishing of the vacuum energy. However, this can be accomplished by imposing on a model additional global symmetries.

This is due to the difference in the number of degrees of freedom for massive and massless spin-2 fields. In our case this difference is very transparent, KK gravitons at each mass level "eat up" three extra degrees of freedom of $g_{5\mu}$ and g_{55} components of the higher dimensional metric ("graviphotons" and "graviscalars" respectively).

Since we choose a model in which there is no normalizable-zero mode, the whole answer is given by the bulk continuum. Let us show that the 4D gravity which is obtained in this way cannot reproduce observable effects of General Relativity. Let us first consider the Newtonian limit. In this case, we take two static point-like sources

$$T_{\mu\nu}(x) = m_1 \delta_{\mu 0}\delta_{\nu 0}\delta(\vec{x}), \quad T'_{\mu\nu}(x') = m_2 \delta_{\mu 0}\delta_{\nu 0}\delta(\vec{x'} - \vec{r}) \ . \tag{16}$$

For this setup the bulk graviton exchange gives

$$\frac{2}{3}\ m_1 m_2\ G_5 \int\ dm\ \frac{e^{-mr}}{r}\ |\psi_m(0)|^2 \ . \tag{17}$$

Since the leading behavior of the integral for the particular case at hand is $1/r$,

$$\int\ dm\ \frac{e^{-mr}}{r}\ |\psi_m(0)|^2 \sim \frac{a}{r} + ..., \tag{18}$$

the correct Newtonian limit may be reproduced (a is some normalization constant). On the other hand, since the exchange of one normalizable massless graviton would give

$$G_N \frac{1}{2}\frac{m_1 m_2}{r}\ , \tag{19}$$

we have to set

$$a\ G_5\ =\ \frac{3\ G_N}{4}\ . \tag{20}$$

This identification provides the correct Newtonian potential for static sources. So far so good. Unfortunately, the problem arises when one tries to account for moving sources. To see this let us take one of the sources to be a moving point-like particle of mass m_2 and proper time τ. The energy-momentum tensor for this particle is written as:

$$T'_{\mu\nu}(x')\ =\ m_2 \int\ d\tau\ \dot{x_\mu}\ \dot{x_\nu}\ \delta(x' - x(\tau))\ . \tag{21}$$

The result of the bulk graviton exchange then gives

$$G_5\ m_1 m_2 \int\ d\tau\ (\dot{x_0}\dot{x_0} - \frac{1}{3}\dot{x_\mu}\dot{x^\mu}) \int\ dm\ \frac{e^{-mr(\tau)}}{r(\tau)}\ |\psi_m(0)|^2\ , \tag{22}$$

where $r = \vec{x}(\tau)$. With the identification (20), in the leading order this yields

$$\frac{3}{4} G_N m_1 m_2 \int d\tau (\dot{x_0}\dot{x_0} - \frac{1}{3}\dot{x_\mu}\dot{x^\mu})\frac{1}{r(\tau)}\ . \tag{23}$$

On the other hand the exchange of a normalizable graviton zero-mode produces the following result

$$G_N m_1 m_2 \int d\tau (\dot{x_0}\dot{x_0} - \frac{1}{2}\dot{x_\mu}\dot{x^\mu})\frac{1}{r(\tau)} \ . \tag{24}$$

This shows the discrepancy between the predictions of the two theories. In particular, the same procedure applied to the problem of bending of light by the Sun gives the discrepancy by the factor 3/4. Indeed, for the bending of light in the gravitational field of the Sun, the tree-level bulk graviton exchange gives:

$$G_5 \ M_{\rm Sun} \ T_{00}(k,q,\epsilon_\mu,\epsilon'_\nu) \int \ dm \ \frac{\delta(k_0-q_0)}{(k-q)^2-m^2-i\varepsilon} \ |\psi_m(0)|^2 \ \simeq \\ -\frac{3}{4}\frac{G_N \ M_{\rm Sun} \ T_{00}\delta(k_0-q_0)}{|\vec{k}-\vec{q}|^2} \ + \ldots \ , \tag{25}$$

where $T_{00}(k,q,\epsilon_\mu,\epsilon'_\nu)$ is the component of the energy-momentum tensor for photons in the momentum representation, and k (ϵ_μ) and q (ϵ'_ν) are the momenta(polarizations) of initial and final photons. This is just 3/4 of the result of the 4D massless graviton exchange.

Summarizing, we have shown that in theories with truly infinite extra dimensions the correct four-dimensional Newtonian gravity can be obtained at intermediate distances due to a metastable resonance graviton. This description is complementary to the exact summation of continuum modes. Due to the finite lifetime of the resonance the laws of gravity are modified at large distances. This would give rise to interesting cosmological consequences. Moreover, these models could allow to preserve bulk supersymmetry while it is completely broken on a brane. Unfortunately, these models encounter a number of phenomenological difficulties. The effects of additional polarization degrees of freedom of massive gravitons survive even in the massless limit and lead to substantial discrepancies with the predictions of General Relativity. It might be possible to cure these discrepancies by introducing new very unconventional interactions. The addition of dilaton-type scalars coupled to T^μ_μ seems to make things worse.

Acknowledgments

We wish to thank Ian Kogan and Valery Rubakov for comments. The work of G.D. is supported in part by David and Lucile Packard Foundation Fellowship for Science and Engineering and by Alfred P. Sloan Research Fellowship. That of G.G. is supported by NSF grant PHY-94-23002. M.P. is supported in part by NSF grant PHY-9722083.

Note added

After this paper was prepared for submission the work [10] appeared. The authors of this work have also realized that metastable gravitons can be responsible for the $1/r$ law in theories with infinite extra dimensions. However, the generic

phenomenological difficulties of this class of theories which is a crucial part of our work have not been addressed in [10].

After this work appeared on the net, we were informed by V. Rubakov that in the revised version of Ref. [6] the role of a metastable graviton was also elucidated and its decay width was calculated.

Appendix

Below we show the presence of a resonance in a system with an infinite extra dimension. The particular example which we consider is the one studied in Ref. [6]. The five-dimensional interval defining the background and four-dimensional graviton fluctuations is set as follows (here we choose to use a non-conformaly flat metric in order to be consistent with the conventions of [6]):

$$ds^2 \;=\; [A(y)\eta_{\mu\nu} \;+\; h_{\mu\nu}(x,y)]\, dx^\mu dx^\nu \;-\; dy^2 \;. \tag{26}$$

There is a 3-brane with positive tension T which is located at $y = 0$. In addition, there are two 3-branes with equal negative tensions $-T/2$ located at a distance y_c to the left and right of the positive-tension brane. For $|y| < y_c$ the space is AdS_5 and the warp-factor is normalized as $A(y) = \exp(-2Hy)$. Furthermore, for $|y| > y_c$, the space becomes Minkowskian and the corresponding warp-factor is a constant, $c^2 \equiv \exp(-2Hy_c)$. Thus, at large distances, i.e. $|y| >> y_c$, the system reduces to a single tensionless brane embedded in five-dimensional Minkowski space-time. For simplicity of presentation in what follows we will deal with the positive semi-axis only, i.e., $y \geq 0$ (the negative part of the whole y axis is restored by reflection symmetry). Choosing the traceless covariant gauge for graviton fluctuations ($h^\mu_\mu = 0,\;\; \partial^\mu h_{\mu\nu} = 0$) the Einstein equations take the form:

$$\begin{aligned} \Psi'' \;-\; 4H^2\Psi \;+\; m^2 e^{2Hy}\Psi \;&=\; 0, \qquad 0 < y < y_c \;, \\ \Psi'' \;+\; \frac{m^2}{c^2}\Psi \;&=\; 0, \qquad y > y_c \;. \end{aligned} \tag{27}$$

Where we have introduced the y dependent part of the fluctuations as follows $h(x,y) \equiv \Psi(y)\exp(ipx)$. Furthermore, the mass-shell condition for graviton fluctuations is defined as $p^2 = m^2$. The equations presented above should be accompanied by the Israel matching conditions at the points where the branes are located. For the particular case at hand these conditions take the form [6]

$$\begin{aligned} \Psi' \;+\; 2H\Psi \;&=\; 0, \qquad y = 0 \;; \\ \Psi'|_{\mathrm{jump}} \;&=\; 2H\Psi, \qquad y = y_c \;. \end{aligned} \tag{28}$$

The solutions to equations (27) are combinations of Bessel functions for $0 < y < y_c$, and exponentials for $y > y_c$:

$$\Psi_m\,(y) \;=\; A_m\; J_2\left(\frac{m}{H}e^{Hy}\right) \;+\; B_m\; N_2\left(\frac{m}{H}e^{Hy}\right) \;, \qquad 0 < y < y_c \;, \tag{29}$$

$$\Psi_m(y) \;=\; C_m \exp\left(i\frac{m}{c}(y-y_c)\right) \;+\; D_m \exp\left(-i\frac{m}{c}(y-y_c)\right) \;, \quad y > y_c \;. \quad (30)$$

The constant coefficients $A_m,\ B_m,\ C_m$ and D_m are determined by using the matching conditions (28) (along with the normalization equation). The presence of a resonance state requires that the coefficient of the incoming wave in the solution (D_m in this case) vanishes at a point in the complex m plane. This determines a resonance. Calculating D_m and putting $D_m = 0$ one finds:

$$K_1(\rho)\; I_2\left(\rho e^{Hy}\right) \;+\; I_1(\rho)\; K_2\left(\rho e^{Hy}\right) \;=\; I_1(\rho)\; K_1\left(\rho e^{Hy}\right) \;-\; K_1(\rho)\; I_1\left(\rho e^{Hy}\right) \;, (31)$$

where we have introduced a new variable $\rho \equiv -im/H$. This relation can be solved for small values of ρ. The result is $\rho \;\simeq\; -2\exp\left(-3Hy_c\right)$. Therefore, the resonance width is proportional to

$$\Gamma \;\propto\; H \;\exp\left(-3Hy_c\right) \;. \quad (32)$$

In the limit $y_c \to \infty$, the resonance width goes to zero and one recovers a zero-mode graviton localized on a positive tension brane [2].

References

[1] N. Arkani-Hamed, S. Dimopoulos, D. Dvali, Phys. Lett. **B429**, 263 (1998); [hep-ph/9803315]; Phys. Rev. **D59**, 0860 (1999); [hep-ph/9807344].

[2] L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); [hep-ph/9905221]; Phys. Rev. Lett. **83**, 4690 (1999); [hep-th/9906064].

[3] M. Visser, Phys. Lett. **B159**, 22 (1985); [hep-th/9910093].

[4] M. Gogberashvili, [hep-ph/9812296]; [hep-ph/9812365].

[5] N. Arkani-Hamed, S. Dimopoulos, D. Dvali, N. Kaloper, [hep-th/9907209].

[6] R. Gregory, V.A. Rubakov, S.M. Sibiryakov, [hep-th/0002072].

[7] H. van Dam, M. Veltman, Nucl. Phys. **B22**, 397 (1970);
V.I. Zakharov, JETP Lett. **12**, 312 (1970).

[8] I.I. Kogan, S. Mouslopoulos, A. Papazoglou, G.G. Ross, J. Santiago, [hep-th/9912552].

[9] G. Dvali, M. Shifman, Nucl. Phys. **B504**, 127 (1997).

[10] C. Csáki, J. Erlich, T.J. Hollowood, [hep-th/0002161].